# Uniform Kernelization Complexity of Hitting Forbidden Minors

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Algorithmic Graph Theory
on the
Adriatic Coast

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Joint work with Bart Jansen, Daniel Lokshtanov, and Saket Saurabh

# F-Minor-Free Deletion

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| Parameter: | $k$. |
| Question: | Does there exist a set $S \subseteq V(G)$ such that $\lvert S \rvert \leq k$ and $G \setminus S$ doesn't contain any of the graphs in $\mathcal{F}$ as a minor? |

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Examples:

- Vertex Cover
- Feedback Vertex Set
- Vertex Planarization
- Treewidth-$\eta$ Deletion
- Treedepth-$\eta$ Deletion

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| $f(\mathcal{F}) k^{O(g(\mathcal{F}))}$ kernel | [Fomin et al, FOCS 2012] |
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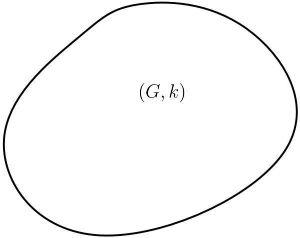
| $k^{\mathcal{O}(\Delta(\mathcal{F}))}$ kernel<br>$k$ is the vertex cover of the input | [Fomin et al, JCSS 2014] |

# Open problem

Can we find a kernel for PLANAR F-MINOR-FREE DELETION of size $f(\mathcal{F})k^c$?

# Kernelization



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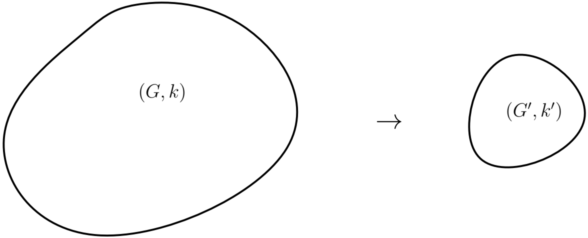

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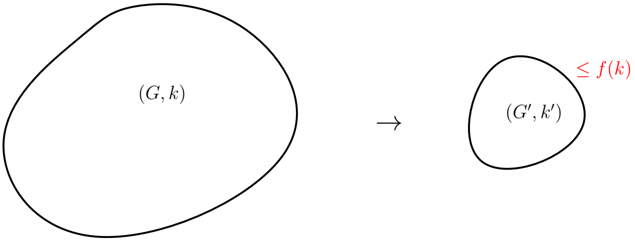

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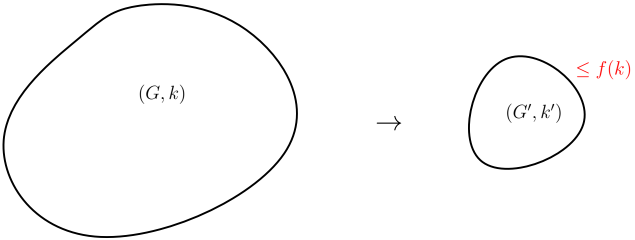



## Kernelization

A *kernelization algorithm* is a polynomial time algorithm that given as input an instance $(G, k)$ to the problem outputs an equivalent instance $(G', k')$ with $k' \leq f(k)$ and $|V(G')| + |E(G')| \leq f(k)$.

# Our Results (Lower Bounds)

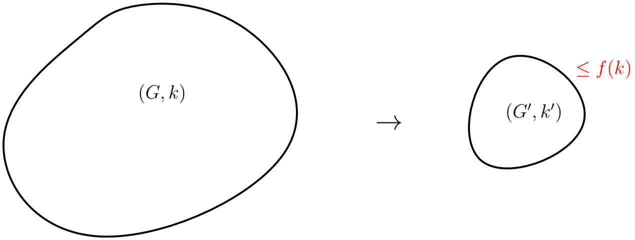
Let $d \geq 3$ be a fixed integer and $\epsilon > 0$.

If the parameterization by solution size $k$ of one of the problems

1. $\{K_{d+1}\}$-Minor-Free Deletion,
2. $\{K_{d+1}, P_{4d}\}$-Minor-Free Deletion, and
3. Treewidth-$(d-1)$ Deletion

admits a kernel with $\mathcal{O}(k^{\frac{d}{4}-\epsilon})$ vertices, then NP $\subseteq$ coNP/poly.

# Compression



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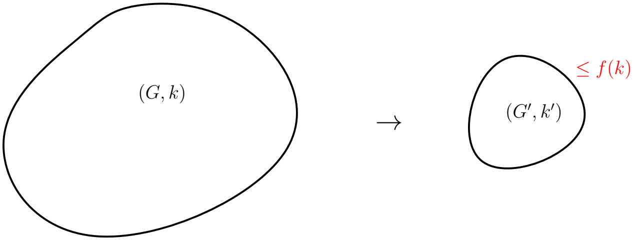
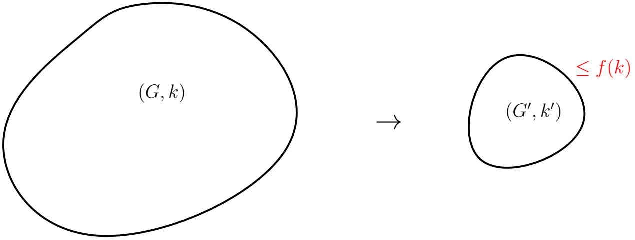



## Compression

A *compression algorithm* is a polynomial time algorithm that given as input an instance $(G, k)$ of a problem $P$ outputs an equivalent instance $(G', k')$ of a problem $Q$ with $k' \leq f(k)$ and $|V(G')| + |E(G')| \leq f(k)$.

# Degree-$d$ polynomial-parameter transformation




## Degree-$d$ polynomial-parameter transformation

A *degree-d polynomial-parameter transformation* is a polynomial time algorithm that given as input an instance $(G, k)$ of a problem $P$ outputs an equivalent instance $(G', k')$ of a problem $Q$ with $k' \in \mathcal{O}(k^d)$.

# Exact $d$-Uniform Set Cover

Given:

- A finite set $U$ of size $n$,
- an integer $k$,
- a family $\mathcal{F} \subseteq 2^U$ of sets of size $d$

Asked:

- $\mathcal{F}' \subseteq \mathcal{F}$ of $k$ sets such that each element of $U$ is contained in exactly one set of $\mathcal{F}'$ .

**Theorem (Dell and Marx, SODA 2012)**

*For every fixed $d \geq 3$ and $\epsilon > 0$, there is no compression of size $\mathcal{O}(k^{d-\epsilon})$ for* Exact $d$-Uniform Set Cover, *unless* NP $\subseteq$ coNP/poly.

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# The reduction

$U = \{1, 2, \ldots, n\}$ and $\mathcal{F}$. We construct $G$ :

$$\begin{array}{cccc}
\circ\, v_{1,1} & \circ & \cdots & \circ\, v_{1,k} \\
\circ\, v_{2,1} & \circ & \cdots & \circ\, v_{2,k} \\
\circ\, v_{3,1} & \circ & \cdots & \circ\, v_{3,k} \\
\circ\, v_{4,1} & \circ & \cdots & \circ\, v_{4,k} \\
\vdots & \vdots & & \vdots \\
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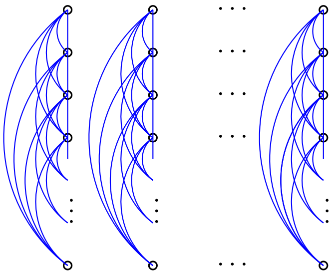

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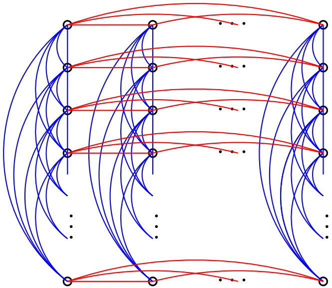

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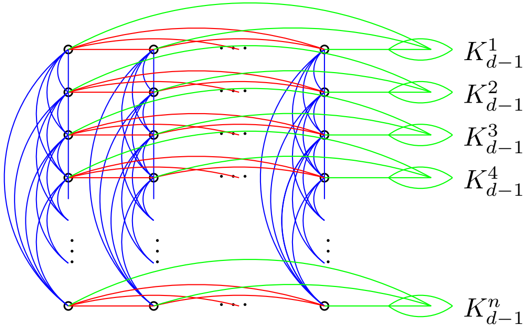

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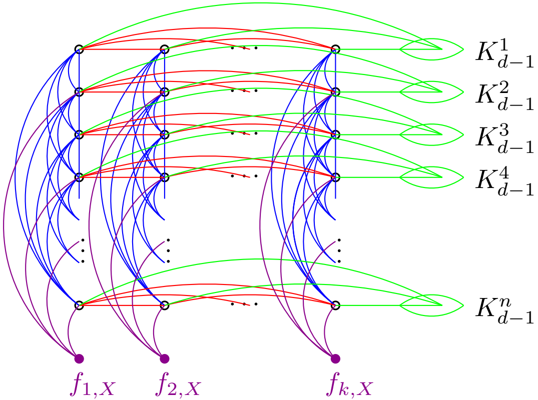


for every $X \in \binom{n}{d} \setminus \mathcal{F}$.

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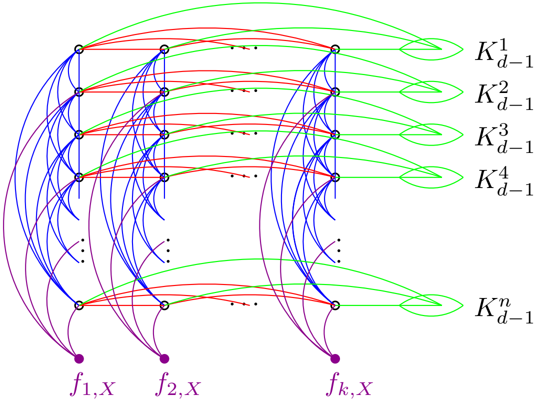


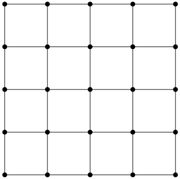

for every $X \in \binom{n}{d} \setminus \mathcal{F}$. Finally, let $k' = k(n-d)$.

# Our Results (TREEDEPTH-$\eta$ DELETION)

**Theorem**

*For each fixed $\eta$, TREEDEPTH-$\eta$-DELETION has a polynomial kernel with $\mathcal{O}(k^6)$ vertices: an instance $(G, k)$ can be efficiently reduced to an equivalent instance $(G', k)$ with $2^{\mathcal{O}(\eta^2)} k^6$ vertices.*

# Why look into TREEDEPTH-$\eta$-DELETION?

| Excluded Graph | Bounded Parameter |
|---|---|
|  | Treewidth |

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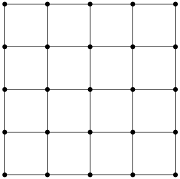
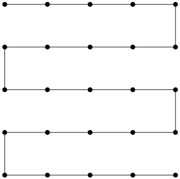
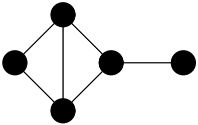
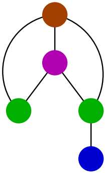
|  | Treewidth |
|  | Treedepth |

# Treedepth

## Definition

**td**$(G) \leq k$ if and only if there exists a rooted forest $F$ of height at most k such that $G \subseteq$ **clos**$(F)$.





The closure of a rooted forest $F$ is obtained the graph **clos**$(F)$ obtained from $F$ after adding edges between each vertex and its proper ancestors.

# Properties of Treedepth

- $\mathcal{D}_{\eta}$ is minor-closed.

- The class $\mathcal{F}_{\eta} = \mathbf{obs}(\mathcal{D}_{\eta})$ consisting of the minor-obstructions of $\mathcal{D}_{\eta}$ is finite. Thus TREEDEPTH-$\eta$ DELETION is equivalent to $\mathcal{F}_{\eta}$-MINOR-FREE DELETION.

- In particular, $P_{2^{\eta}} \in \mathcal{F}_{\eta}$ and TREEDEPTH-$\eta$ DELETION is equivalent to PLANAR $\mathcal{F}_{\eta}$-MINOR-FREE DELETION.

# Reduction Rule 1 (Vertex Removal)

## Lemma

*Let $(G, k)$ be an instance of* Treedepth-$\eta$ Deletion *and let $\ell$ be an integer. Let $S \subseteq V(G)$ such that $N_G(S)$ is a clique and $\mathbf{td}(G[S]) \leq \eta$. For every $v \in N_G(S)$, let $X_1^v, \ldots, X_{\ell+\eta}^v \subseteq V(G)$ induce connected subgraphs of $G$ such that:*

1. $\forall v \in N_G(S), \forall i \in [\ell + \eta]$: $\mathbf{td}(G[X_i^v]) \geq \mathbf{td}(G[S])$ *and* $v \in N_G(X_i^v)$,
2. $\forall v \in N_G(S)$, *the sets* $X_1^v, \ldots, X_{\ell+\eta}^v$ *are pairwise disjoint and disjoint from $S$, and*
3. $G - S$ *has a minimum treedepth-$\eta$ modulator containing $\leq \ell$ vertices of $\mathcal{X}$,*

*where $\mathcal{X} := \bigcup_{v \in N_G(S)} \bigcup_{i \in [\ell+\eta]} X_j^v$. Then $(G, k)$ is equivalent to the instance $(G - S, k)$.*

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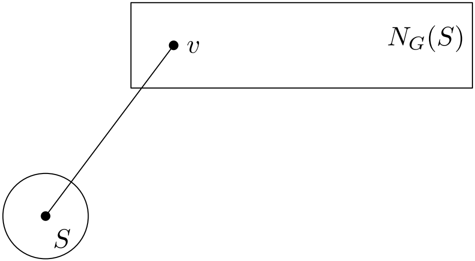

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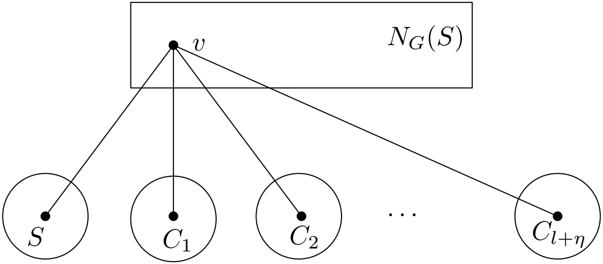


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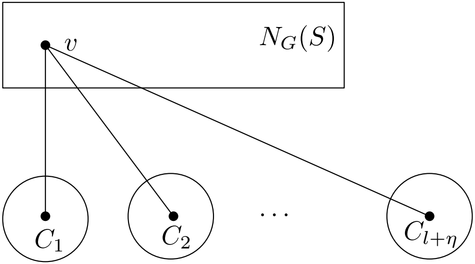


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# Reduction Rule 2 (Edge Addition)

**Lemma**

*Let $(G, k)$ be an instance of* TREEDEPTH-$\eta$ DELETION *and let $\ell$ be an integer. Let $X \subseteq V(G)$ and let $\{u, v\} \in \binom{V(G)}{2} \setminus E(G)$. If the following conditions hold:*

1. *the graph $G[X \cup \{u, v\}]$ contains at least $\ell + \eta$ internally vertex-disjoint paths between $u$ and $v$, and*
2. *$G$ has a minimum treedepth-$\eta$ modulator containing $\leq \ell$ vertices of $X$,*

*then $(G, k)$ is equivalent to the instance $(G + uv, k)$ obtained by adding the edge $uv$.*

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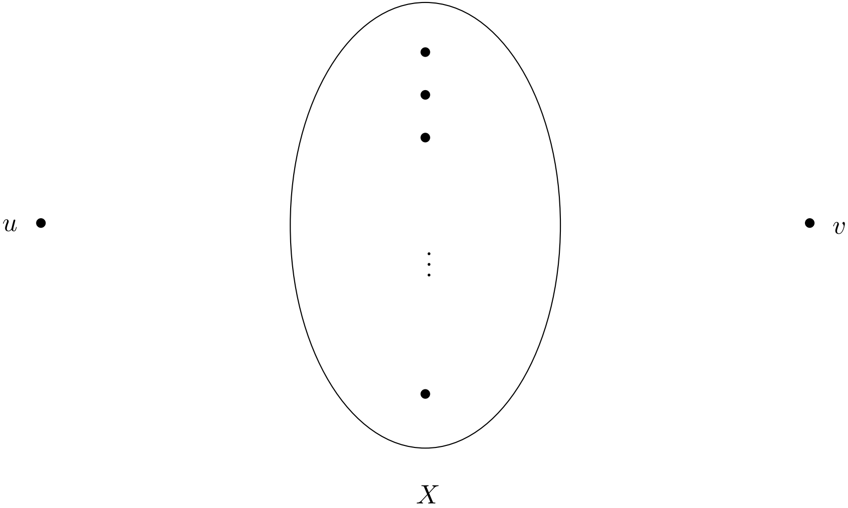

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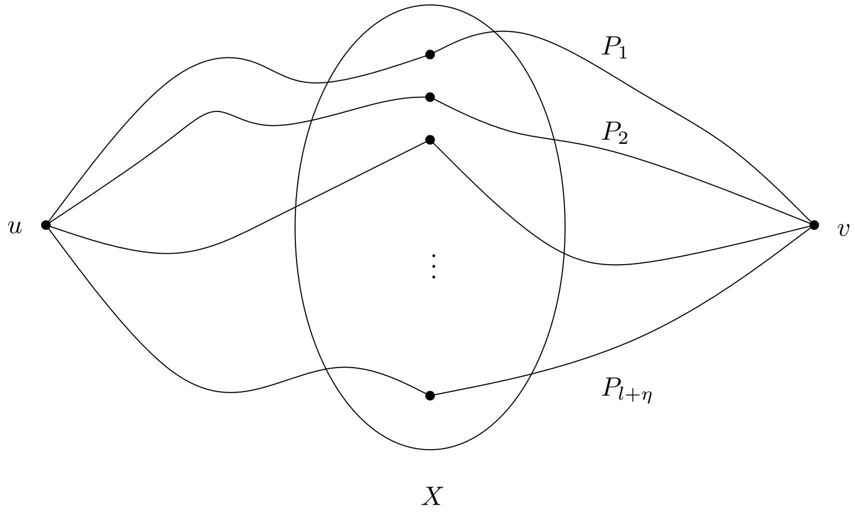

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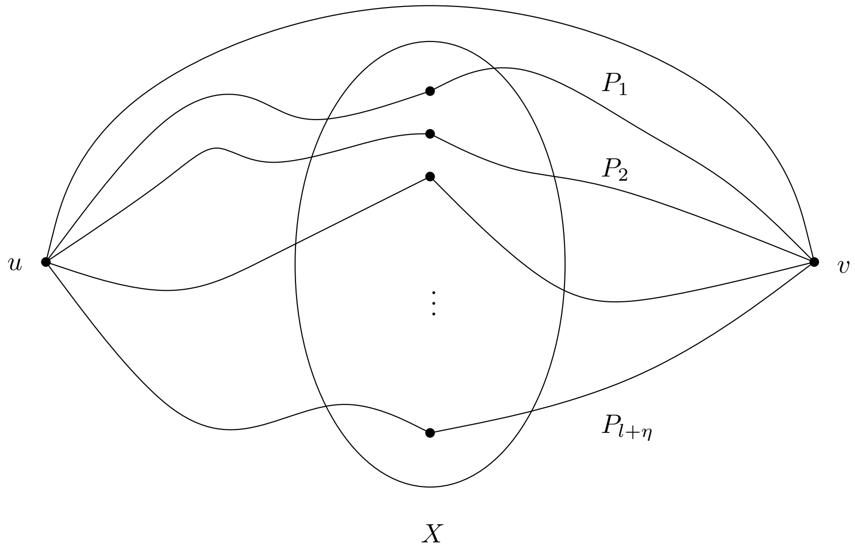

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1. $\forall i \in [\ell + \eta]$: $\mathbf{td}(G[X_i]) \geq \mathbf{td}(G[S])$ *and* $v \in N_G(X_i)$,
2. *the sets $X_1, \ldots, X_{\ell+\eta}$ are pairwise disjoint and disjoint from $S$, and*
3. *any graph obtained from $G$ by removing edges between $v$ and $S$ has a minimum treedepth-$\eta$ modulator containing $\leq \ell$ vertices of $\mathcal{X}$,*

*where $\mathcal{X} := \bigcup_{i \in [\ell+\eta]} X_j$. Then $(G, k)$ is equivalent to the instance $(G', k)$, where $G'$ is obtained from $G$ by removing all edges between $v$ and $S$.*

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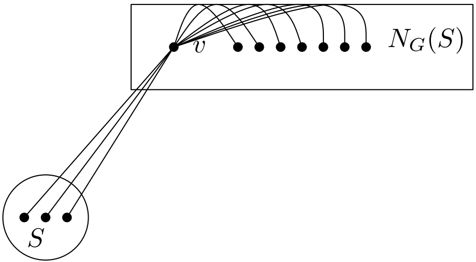

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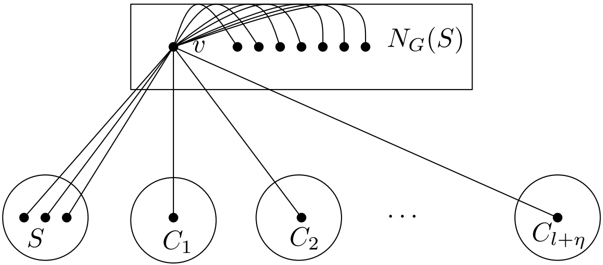


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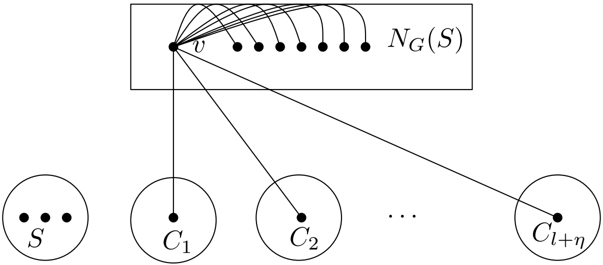


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# Approximation Algorithm

Together with the Reduction Rules we need the following algorithm to obtain the kernel for $\textsc{Treedepth-}\eta$ $\textsc{Deletion}$.

**Lemma**

*Fix $\eta \in \mathbb{N}$. Given a graph $G$, one can in polynomial time compute a subset $S \subseteq V(G)$ such that $\mathbf{td}(G - S) \leq \eta$ and $|S|$ is at most $2^\eta$ times the size of an optimal treedepth-$\eta$ modulator of $G$.*

[Gajarsky et al., ESA 2013]

# Overview

The kernel is obtained in two phases:

- Decomposition of $(G, k)$ to an equivalent instance $(G', k')$
- Application of Reduction Rules to obtain an equivalent instance $(G'', k)$ of reduced size.

## Graph Decomposition

Let $(G, k)$ be an of Treedepth-$\eta$ Deletion.

# Graph Decomposition

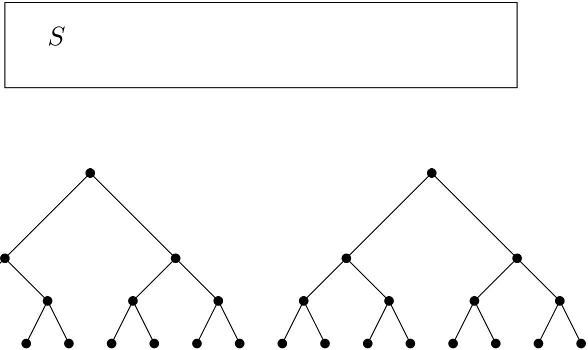
Let $(G, k)$ be an of $\textsc{Treedepth-}\eta$ $\textsc{Deletion}$.

- For every $p, q \in V(G)$ where $\{p, q\} \notin E(G)$, if there exist $k + \eta$ internally disjoint $(p, q)$-paths add $\{p, q\} \in E(G)$.

# Graph Decomposition

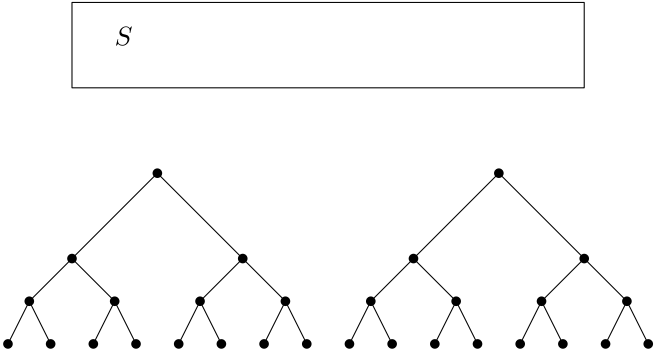
Let $(G, k)$ be an of TREEDEPTH-$\eta$ DELETION.

- For every $p, q \in V(G)$ where $\{p, q\} \notin E(G)$, if there exist $k + \eta$ internally disjoint $(p, q)$-paths add $\{p, q\} \in E(G)$.
- Using the approximation algorithm to find $S$ such that $\mathbf{td}(G \setminus S) \leq \eta$. Then $|S| \leq 2^{\eta}k$. Let $F$ denote the forest where $G \setminus S \subseteq \mathbf{clos}(F)$.



# Graph Decomposition

- For every $p, q \in S$ where $\{p, q\} \notin E(G)$ find a minimum $(p, q)$-separator $Y_{p,q}$ with $|Y_{p,q}| < k + \eta$.



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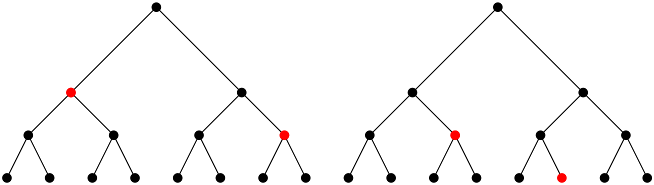
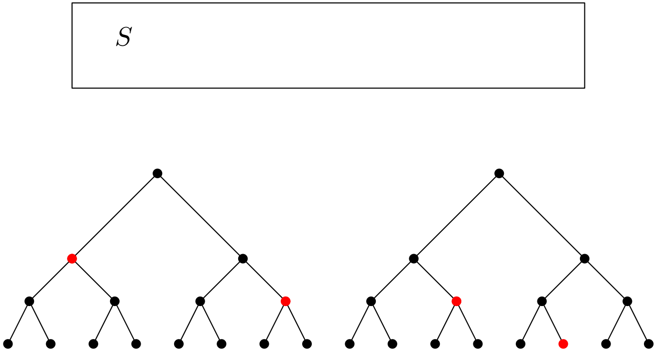


# Graph Decomposition

- Let $Y = \bigcup_{\{p,q\} \notin E(G)} Y_{p,q}$. For every $y \in Y$ add in $Y$ all the proper ancestors of $y$.



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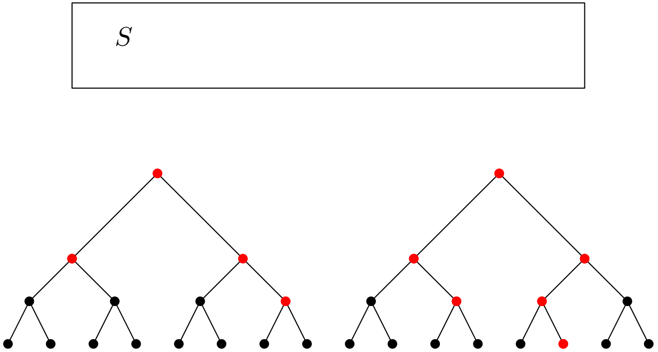

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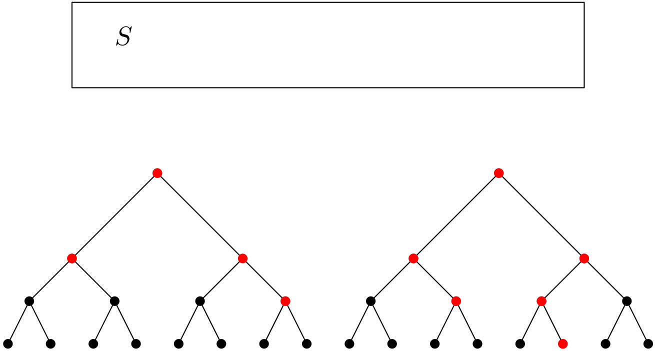
- Let $\mathcal{T}$ be the roots of the forest $F'$ obtained from $F$ after removing $Y$.



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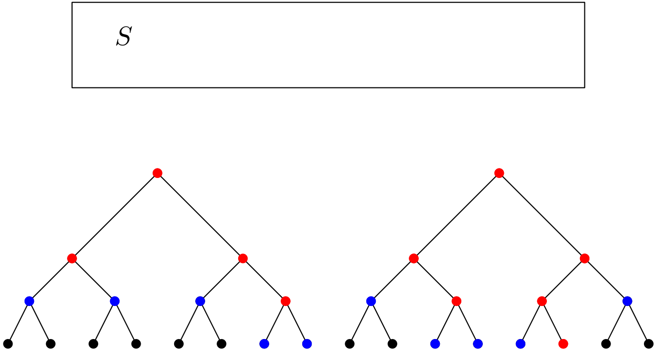
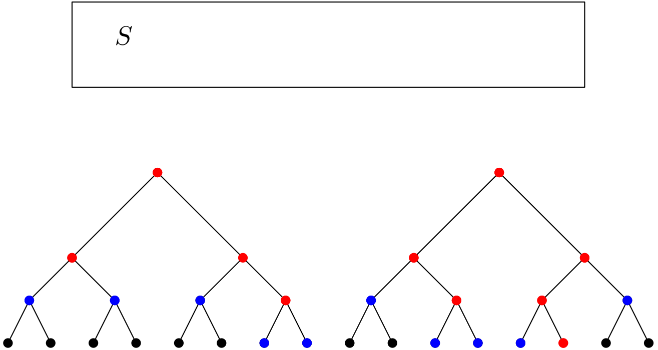


# Graph Decomposition

- While there is a vertex $u_0 \in \mathcal{T}$, where for the set $V(F_{u_0})$ Reduction Rule 1 is applicable, apply Reduction Rule 1 and remove $u_0$ from $\mathcal{T}$.



# Properties of the New Instance

Let $(G, k)$ be an instance of TREEDEPTH-$\eta$ DELETION.

1. $(G, k)$ is equivalent to $(G', k)$
2. $|S| \leq 2^{\eta} \cdot k$.
3. $|Y| \leq \eta(2^{\eta} \cdot k)^2 \cdot (k + \eta)$.
4. For every $u \in V(F') \setminus Y$ the graph $G'[F'_u]$ is connected.

# Properties of the New Instance

Let $(G, k)$ be an instance of TREEDEPTH-$\eta$ DELETION.

1. Let $\mathcal{T}' := \{u \in F' - Y \mid u \text{ is a root or } \pi(u) \in Y\}$. The vertex sets of the connected components of $G' - (S \cup Y)$ are exactly the vertex sets of the subtrees of $F'$ rooted at members of $\mathcal{T}'$.
2. For every connected component $C$ of $G' - (S \cup Y)$, the set $N_{G'}(C) \cap S$ is a clique and for every minimal treedepth-$\eta$ modulator $Z$, $Z \cap V(C) \leq 2\eta$.
3. The number of connected components of $G' - (S \cup Y)$ is at most $(|S| + |Y| + |S|^2 + |S| \cdot |Y| + \eta \cdot |Y|) \cdot (\eta + k)$.

# Bounding the Size of the Connected Components

Let $T$ be the tree in $F'$ with $v$ as root.

- While there are $p, q \in N(T_v) \cup \{v\}$ with $pq \notin E(G)$, joined by $3\eta$ internally vertex disjoint paths in $V(T_v) \cup \{p, q\}$, add $pq \in E(G)$. [Edge addition]

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- While there exist distinct children $c_0, \ldots, c_{3\eta}$ of $v$ s.t. $s \in N_S(c_0) \neq \emptyset$ and $N_G[T_{c_0}] \subseteq N_G[s]$, $\mathbf{td}(G[T_{c_i}]) \geq G[T_{c_0}]$ and $s \in N_G[T_{c_i}]$, remove the edges $vu$, $u \in V(T_{c_0})$ [Edge Deletions]

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- While there exists a child $c^*$ of $v$ s.t. $N_G[T_{c^*}]$ is a clique, and for every $w \in N_G[T_{c^*}]$ there are $3\eta$ distinct children of $v$, $c_i^w \neq c^*$, $i \in [3\eta]$ with $\mathbf{td}(G[T_{c_i^w}]) \geq \mathbf{td}(G[T_{c^*}])$ and $w \in N_G[T_{c_i^w}]$, remove $T_{c^*}$ from $F$ and from $G$. [Vertex deletions]

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- Call the algorithm for every remaining child of $v$.

# The Size of the Reduced Components

By counting arguments that depend on the properties of the connected components

$$|C| \leq \eta \cdot (2 \cdot 3\eta \cdot 2^{\eta})^{\eta}(|S| + 1)$$

This implies that

$$V(G') \in \mathcal{O}(k^6)$$

# Summarizing

- When $\mathcal{F}$ is one the following three graph classes
  - $\{K_{d+1}\}$
  - $\{K_{d+1}, P_{4d}\}$
  - Obstructions to Treewidth-$(d-1)$

  the size of the kernel parameterized by the solution size is $\Omega(k^{\frac{d}{4}} - \epsilon)$ unless NP $\subseteq$ coNP/poly.

- The problem $\textsc{Treedepth-}\eta$ $\textsc{Deletion}$ parameterized by the solution size admits a kernel with $2^{\mathcal{O}(\eta^2)}k^6$ vertices.

# Open Problems

- Does there exist a family $\mathcal{F}$ that does not contain planar graphs and admits a polynomial size kernel?

- Is it possible to obtain a dichotomy theorem characterizing for which families $\mathcal{F}$ the problem Planar $\mathcal{F}$-Minor-Free Deletion admits uniformly polynomial kernels?

- Obtain a kernel for Vertex Planarization.

# Thank you for your attention!